Cambridge International AS Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary Level

MATHEMATICS **9709/22**

Paper 2 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1	Either		
	State or imply non-modular inequality $(3x-5)^2 < 4x^2$ or corresponding equation or pair of linear equations	B1	SC: Common error $(3x-5)^2 < 2x^2$
	Attempt solution of 3-term quadratic equation or solution of 2 linear equations	M1	
	Obtain critical values 1 and 5	A1	Critical values $\frac{15 \pm 5\sqrt{2}}{7}$ or 3.15, 1.13 allow B1
	State correct answer $1 < x < 5$	A1	$\frac{15-5\sqrt{2}}{7} < x < \frac{15+5\sqrt{2}}{7}$ or $1.13 < x < 3.15$ B1 Max 2/4 Allow M1 for $(7x \pm 5)(x \pm 5)$
	Or		
	Obtain $x = 5$ by solving linear equation or inequality or from graphical method or inspection	B1	Allow B1 for 5 seen, maybe in an inequality
	Obtain $x = 1$ similarly	B2	Allow B2 for 1 seen, maybe in an inequality
	State correct answer $1 < x < 5$	B1	
		4	

Question	Answer	Marks	Guidance
2	Recognise 9^x as $(3^x)^2$ or 3^{2x}	B1	May be implied by $3^x\left(3^x+1\right)(=240)$
	Attempt solution of quadratic equation in 3^x	*M1	Perhaps using substitution $u=3^x$
	Obtain, finally, $3^x=15$ only	A1	
	Apply logarithms and use power law for $3^x=k$ where $k>0$	M1	Dependent *M, need to see $x\ln 3=\ln k$, $x=\log_3 k$ oe
	Obtain 2.465	A1	May be done using $9^{\frac{x}{2}}$, same processes
		5	

Question	Answer	Marks	Guidance
3	Differentiate to obtain $10\cos 2x$	B1	
	Differentiate to obtain $-6\sec^2 2x$	B1	
	Equate first derivative to zero and find value for $\cos^3 2x$	M1	
	Use correct process for finding x from $\cos^3 2x=k$	M1	
	Obtain 0.284 nfww	A1	Or greater accuracy
		5	

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Question	Answer	Marks	Guidance
4	Obtain $6ye^{2x}+3e^{2x}\dfrac{dy}{dx}$ as derivative of $3ye^{2x}$	**B1**	Allow unsimplified
	Obtain $2y\dfrac{dy}{dx}$ as derivative of y^2	**B1**	
	Obtain 4 as a derivative of $4x$ and zero as a derivative of 10	**B1**	Dependent B mark, must have at least one of the two previous B marks
	Substitute 0 and 2 to find gradient of curve	**M1**	Dependent on at least one B1
	Obtain $-\frac{16}{7}$ or -2.29	**A1**	Allow greater accuracy
		5	

Question	Answer	Marks	Guidance
5(i)	Rearrange at least as far as $2x=\ln(...)$	**M1**	Allow if in terms of p, need to see y equated to 0
	Obtain $x=\frac{1}{2}\ln(1.6x^2+4)$	**A1**	AG; necessary detail needed
		2	

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Question	Answer	Marks	Guidance
5(ii)	Either		
	Consider sign of $x-\frac{1}{2}\ln(1.6x^2+4)$ for 0.75 and 0.85 or equivalent	**M1**	Need to see substitution of numbers
	Obtain –0.04 and 0.03 or equivalents and justify conclusion	**A1**	AG; necessary detail needed, change of sign or equivalent must be mentioned
	Or		
	Consider sign of $5e^{2x}-8x^2-20$ for 0.75 and 0.85	**M1**	Need to see substitution of numbers
	Obtain –2.09... and 1.58… or equivalents and justify conclusion	**A1**	AG; necessary detail needed, change of sign or equivalent must be mentioned
		2	
5(iii)	Use iteration process correctly at least once	**M1**	Starting with value such that iterations converge to correct values
	Obtain final value 0.80956	**A1**	Must be 5sf for the final answer
	Show sufficient iterations to justify value or show sign change in interval (0.809555, 0.809565)	**A1**	
		3	

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Question	Answer	Marks	Guidance
5(iv)	Obtain first derivative $10e^{2x} - 16x$	B1	
	Substitute value from iteration to find gradient, must be in the form $pe^{2x} + qx$	M1	
	Obtain 37.5	A1	Or greater accuracy, allow awrt 37.5 from use of $x = 0.8096,\ \ 0.80955$ oe
		3	

Question	Answer	Marks	Guidance
6(a)	Integrate to obtain form $k\ln(3x+2)$	*M1	Condone poor use of brackets if recovered later
	Obtain correct $4\ln(3x+2)$	A1	
	Substitute limits correctly	M1	Dependent *M, must see $k\ln 20 - k\ln 5$ oe
	Apply relevant logarithm properties correctly	M1	Dependent *M, do not allow $\frac{4\ln 20}{4\ln 5}$ oe, must be using both the subtraction and power laws correctly
	Obtain $\ln 256$ nfww	A1	AG; necessary detail needed
		5	

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Question	Answer	Marks	Guidance
6(b)	Use identity to obtain $4(1-\cos 2x)$ oe	**B1**	
	Use identity to obtain $\sec^2 2x - 1$	**B1**	
	Integrate to obtain form $k_1 x + k_2 \sin 2x + k_3 \tan 2x$	***M1**	Allow M1 if integrand contains $p\cos 2x + q\sec^2 2x$ and no other trig terms
	Obtain correct $3x - 2\sin 2x + \frac{1}{2}\tan 2x$	**A1**	
	Apply limits correctly retaining exactness	**M1**	Dependent *M, allow $\sin\frac{\pi}{3}$, $\tan\frac{\pi}{3}$
	Obtain $\frac{1}{2}\pi - \frac{1}{2}\sqrt{3}$ or exact equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
7(i)	Substitute $-\frac{3}{2}$ and simplify	**M1**	Allow use of identity assuming a factor of $2x+3$ to obtain a quadratic factor. Need to see use of 4 equations to verify quadratic for M1, A1 for conclusion. Allow verification by expansion. Allow use of identity including a remainder to obtain a quadratic factor and a remainder of zero. Need to see use of 4 equations for M1, A1 for conclusion. Allow verification by expansion. Allow use of long division , must reach a remainder of zero for M1
	Obtain $-27+9+15+3$ or equivalent, hence zero and conclude , may have explanation at start of working	**A1**	Need powers of $-\frac{3}{2}$ evaluating for A1 AG; necessary detail needed
		2	
7(ii)	Use $\cos 2\theta = 2\cos^2\theta - 1$	**B1**	
	Simplify $a\cos^2\theta + b = \dfrac{6\cos\theta - 5}{2\cos\theta + 1}$ to polynomial form	**M1**	
	Obtain $8\cos^3\theta + 4\cos^2\theta - 10\cos\theta + 3 = 0$	**A1**	AG; necessary detail needed, must be completely correct with no poor use of brackets for A1
		3	

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Question	Answer	Marks	Guidance
7(iii)	Attempt either division by $2x+3$ and reach partial quotient x^2+kx or use of identity or inspection	*M1	Or equivalent using $\cos\theta$ or c
	Obtain quotient $4x^2-4x+1$	A1	Or equivalent
	Obtain factorised form $(2x+3)(2x-1)^2$	A1	Or equivalent, may be implied by later work
	Solve for $\cos\theta = k$ to find at least one value between 0 and 360	M1	Dependent *M
	Obtain 60 and 300 and no others	A1	SC1: Equation solver used to obtain 60 and 300 and no others, then 5/5 SC2:Equation solver used to obtain 60 then 4/5 SC 3: $\cos\theta = 0.5$, $\left(\cos\theta = -1.5\right)$ seen implies first 3 marks.
		5	